SOLUTION BY ELIJAH SWIFT, University of Vermont.

In the integrand of (1) the radical is to be taken with the plus sign, as $[r^2 + z^2]^{\frac{1}{2}}$ is a length, namely, the radius vector from the origin to the element in question. In evaluating (2), then, we must take the radical with the plus sign. But when r is zero the value of the radical is $\pm z$, so that we must take $+ z$ when z is positive, $- z$ when z is negative. In obtaining (3), however, this value was taken as $+ z$ for all values of z. (3) should read

$$2\pi k\delta \left[\int_0^R \left[\frac{-z}{R} + 1 \right] dz + \int_{-R}^0 \left[\frac{-z}{R} - 1 \right] dz \right] = 0.$$

Also solved by PAUL CAPRON, J. W. CLAWSON, GEORGE PAASWELL, K. P. WILLIAMS, and the PROPOSER.

323. Proposed by CLIFFORD N. MILLS, Brookings, South Dakota.

Two equal bodies are placed on a rough inclined plane, being connected by a light inelastic string; if the coefficients of friction are respectively 1/3 and 1/4, show that they will both be on the point of motion when the inclination of the plane is $\sin^{-1}(7/25)$.

SOLUTION BY H. S. UHLER, Yale University.

In order to obtain the given inclination it is necessary to assume that the string lies in a vertical plane which is perpendicular to the intersection of the inclined plane with any horizontal plane, that it is taut, etc. From simple mechanical considerations it is evident that the body associated with the greater coefficient of friction must be higher up on the incline than its tandem body. Let m and T denote, respectively, the mass of either body and the tension of the string. By resolving the weight (mg) of each body parallel and perpendicular to the incline, and by employing the definition of the coefficient of friction, we find at once that the condition for being on the point of motion is expressed by the equations

$$T + mg \sin \alpha - \tfrac{1}{3}mg \cos \alpha = 0,$$

$$- T + mg \sin \alpha - \tfrac{1}{4}mg \cos \alpha = 0,$$

where α symbolizes the angle of elevation of the inclined plane. By first adding these equations and then dividing through by mg we get

$$2 \sin \alpha - (7/12) \cos \alpha = 0,$$

or

$$\alpha = \tan^{-1}(7/24) = \sin^{-1}(7/25).$$

Also solved by J. ROSENBAUM, HORACE OLSON, H. C. FEEMSTER, C. A. NICKLE, G. PAASWELL, W. C. EELLS, J. A. CAPARO, W. H. THOME, and A. G. RAU.

NUMBER THEORY.

211. (April, 1914.) Proposed by E. T. BELL, University of Washington.

If an odd perfect number exists, the total number of its divisors is a multiple of 2 but not of 4; or, what is the same thing, an odd perfect number must be of the form $p^{2a-1}n^2$ where p is a prime and a is odd.

REMARK BY TRACY A. PIERCE, Harvard University.

Lucas in his *Théorie des Nombres* proved that an odd perfect number must be of the form $(4m + 1)^{4k+1}n^2$, where $4m + 1$ is a prime. See an article by BOURLET in *Nouv. Ann. de Math.*, 1896, pp. 297–312.

208. (March, 1914.) Proposed by E. T. BELL, University of Washington.

If an odd number be perfect it cannot be the sum of two squares.

Remarks by Tracy A. Pierce, Harvard University.

It is well known that every prime of the form $4m + 1$ is the sum of two squares. Using Lucas's form of an odd perfect number (see Problem 211 above) we see that

$$(4m + 1)^{4k+1}n^2 = (x^2 + y^2)[(4m + 1)^{2k}]^2n^2 = (x^2 + y^2)N^2 = X^2 + Y^2,$$

contrary to the proposition as proposed.

229. (March, 1915.) Proposed by WALTER C. EELLS, Whitman College.

If p and q are integers and p is prime and positive, find the condition on q that the equation $p^x = qx$ shall have integral solutions, solve for x, and show that for a special value of p it has two solutions for a certain q, otherwise only one.

I. Solution by Frank Irwin, University of California.

Since p is prime, we must have $x = p^a$, $q = p^b$, where a and b are positive integers or zero (for a). Now $p^{a+b} = qx = p^{p^a}$, so that $b = p^a - a$, and q is necessarily of the form $p^{(p^a-a)}$. This condition is evidently also sufficient.

Given, then, such a q, the exponent $p^a - a$ may be determined; then a, which will give us x, is that number which must be added to this exponent to make it equal to the *next higher* power of p. For no power of p can lie between $p^a - a$ and p^a, since $p^a - a > p^{a-1}$, as may be readily proved, for instance by mathematical induction.

Of the cases that require special investigation, $a = 0$, and $p = 2$ with $a = 1$ or 2, the only one for which, given $p^a - a$, there is more than one solution for a, is the case

$$2^a - a = 1,$$

which has two solutions $a = 0, 1$. There are two solutions of our problem then for the case $p = 2$, $q = 2$, viz., $x = 1, 2$.

II. Solution by the Proposer.

Consider the two functions, $y = p^x$, $y = qx$. For $x = k$ (any integer), $y = p^k$. The slope of line $y = qx$, passing through (k, p^k), is $q = p^k/k$, which is integral if and only if

$$k = p^n \qquad (n = 0, 1, 2, 3, \cdots), \ i.\ e.,\ q = p^{(p^n-n)}.$$

(If $k < 0$, q is fractional since it is then $= 1/kp^k$).

Substituting this value of q in the given equation, it is easily seen that it is satisfied if and only if

$$x = p^n \qquad (n = 0, 1, 2, 3, \cdots).$$

Consider the exponent of p, namely $p^n - n$. We have

$$[p^n - n]_{n=0} = 1, \quad \text{and} \quad [p^n - n]_{n=1} = p - 1.$$

Then $x_1 = p^0$ and $x_2 = p^1$ will be solutions of the given equation if $1 = p - 1$, *i. e.*, if $p = 2$. From the graphs of the exponential function it is easily seen that $y = qx$ can have but one integral intersection if $p \neq 2$, $n_1 \neq 0$, $n_2 \neq 1$.

The equation having two solutions is $2^x = 2x$, of which $x_1 = 1$, $x_2 = 2$.

230. (April, 1915.) Proposed by E. B. ESCOTT, Ann Arbor, Michigan.

Find three numbers such that their sum, the sum of their squares, and the sum of their cubes, shall be a cube.

Note.—W. D. Cairns says this problem, which was proposed in *L'Intermediaire* in 1900, remains unsolved to date, even though it was reprinted in February, 1913.

Remarks by Artemas Martin, Washington, D. C.

The above problem was published in the *Mathematical Visitor*, Vol. I, No. 1 (Erie, Pa., March, 1877), page 6, as No. 9 in a list of "Unsolved Problems." So far as the writer at present knows that was the first publication of the problem and it still remains "unsolved."